Senior Project Proposal:

“Investigating a Correlation between Minimal Surfaces and Relativistic String Dynamics”

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Project Summary

This proposed project consists of a primary target, and several additional prospective goals. The target, which I aim to complete by the midterm deadline and will refer to as “Part I”, is to determine the extent of the correspondence between classical minimal surfaces in Euclidean 3-space, and classical relativistic string solutions (worldsheets) in 2+1 Minkowski space. Additional “Part II” goals involve extending this correspondence to higher dimensional, but still flat, spacetimes; or, exploring a correspondence between solutions in curved (spherical or hyperbolic) spacetimes, and their Lorentzian analogues DeSitter Space and Anti-DeSitter Space, likely while remaining in low dimensions. Due to time constraints, and in the interest of a focused and complete semester project, likely only one of these additional goals would be selected for pursuit during the second half of the semester. Per Professor Comins’ suggestion, each possibility could be explored for a short time to assess feasibility and significance, before a final decision; certainly this decision would be postponed until after the initial planned work in flat 3-space will have been completed.

The premise for a correspondence between string world-sheets and minimal surfaces arises from the equations of motion which describe classical relativistic strings. These equations may be extracted from several Lagrangian formulations; the most convenient of these for this work is the Nambu-Goto action [1] (image from [6]), where X denotes coordinates on the string world-sheet:

$$S = -T \int d^2\sigma \sqrt{-(\dot{X})^2 (X')^2 + (\dot{X} \cdot X')^2} \, .$$

This action can be shown to be proportional to the area traced out by the propagating string, and so the minimization of the action is linked to a minimization of surface area. Minimized local surface area is one of the qualities that defines minimal surfaces, another being zero mean curvature at all points on the surface, which is most easily calculated by principles of differential geometry [2]; thus the relationship under study begins to emerge. Determining the extent to which this relationship holds, including the cardinality of the relationship (e.g. if it is one-to-one) is the primary subject of this project. Furthermore, exposing correspondences between physical properties of the string solutions, such as the flow of momentum, to mathematical qualities of surfaces, such as curvature, has the potential to lead to valuable insights.

Goals

The essential goal of Part I is to establish what characteristics of possible relativistic string solutions in 2+1 Minkowski space cause the worldsheet to satisfy the characteristics of a minimal surface. A series of specific sub-goals to this end are outlined here, each representing one week of work, and in order of intended completion. I have already performed a significant amount of background study, working through examples and problems in an undergraduate string theory text [1] during this past summer.

- Gain sufficient background in basic differential geometry and the theory of minimal surfaces [2][3][4][5].
- As a primer, investigate the two known examples of the correspondence (provided by Professor Kleban at NYU via Professor Comins), namely the helicoid and catenoid.

- Consider general solutions for both closed and open strings lying in a 2-plane, and determine which meet the characteristics of minimal surfaces.
- Consider general minimal surfaces in three dimensions, with the condition that they have some “time-like” axis, anywhere along which a planar cross-section will intersect a single “string-like” curve, either open or closed (topologically either a line segment or a circle).
- Investigate well-known exotic minimal surfaces which do not meet the previous qualification, such as the Scherk and Enneper surfaces [4], to confirm that they cannot correspond to string worldsheets; if in fact they can, appropriate generalizations will be made to the conditions applied in the previous two sub-goals.

It would be premature at this stage to construct sub-goals for the proposed Part II explorations into minimal surfaces in curved or higher-dimensional spaces; however, the same general procedure would likely be followed, of searching from both the string solution side and minimal surface side for conditions that imply correspondence.

Significance

It is natural to ask where this project fits into the greater scheme of current physics, and what effects might follow from a conclusive result. Although string theory is strongly promoted as a promising lead for a theory of quantum gravity [8], it remains somewhat controversial due to a lack of experimental falsifiability with today’s technology. It is noteworthy that these difficulties are not inherently ‘stringy’; they arise due to the tiny scale of the gravitational coupling constant, which any successful Grand Unified Theory must incorporate, and therefore must also struggle with experimentally [7]. Another valid argument by opponents is that any benefit (other than heightened knowledge) to the general populace due to string theory might be centuries away. Along these same lines, Murray Gell-mann reportedly once joked that his research might have received stronger funding had he claimed to be working on a ‘quark bomb’. Yet perhaps the inability to foresee relevant technological advances provides all the more incentive to study string theory, as all the great advances which physics has made possible for mankind came from exploring novel, seemingly far-fetched concepts.

In beginning this project, as with any scientific endeavour, a number of societal considerations must be taken into account:

> **Economic:** Economic impacts of this project, and string theory as a whole in the foreseeable future, are negligible. There is no tangible product being produced, so monetary costs are also negligible, unless a need should arise for especially powerful computing to assist with evaluation of minimal surfaces.
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> **Environmental:** Environmental impacts of this project, and string theory as a whole in the foreseeable future, are negligible. No experimental materials of any kind are being used that could impact the natural world; the only measurable cost would be the use of electricity for computation, which should be very small.
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> **Sustainability:** As this project and its goals are largely mathematical in nature, there are no natural resources at risk of overuse; nor will this project be impacted in any foreseeable way by major sustainability challenges such as climate change or material shortages.
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> **Manufacturability:** As no tangible product is being designed or produced, manufacturability does not impact this project.
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> **Ethical:** The pursuit of knowledge about the natural world certainly violates no ethical principles, and serves to stimulate human curiosity and future investigation. On the

matter of practical scientific ethics, measures must be taken as always to ensure the protection of intellectual property, and proper recognition for resources utilized.

Health and safety: No hazardous materials or procedures are involved with this theoretical investigation, and no safety practices need be specified other than common sense.

Social: Although there are many deep philosophical questions which are intrinsically linked to fundamental physics and therefore string theory, none bear relevance to this project. The average member of society is not likely to impacted in any way by near-future advances in string theory.

Political: Current political discourse has likely not ever included string theory, and any political impacts would remain unlikely until such a time as there are also relevant economic impacts.

It is as yet unclear specifically how the flat, 3-dimensional case which is first to be explored might significantly contribute to the development of string theory. Such a simplified case is unlikely to result in any kind of “realistic” physical theory, but the classification of string solutions based on minimal surface solution techniques could lay a mathematical base for future research on the more refined string theories which have shown so much promise.

Source Material

Only some of the literature sources referenced below are cited specifically in this proposal; but each one, in part or in whole, has already proven useful to the development of this project idea, and will likely be referenced again in future reports of progress.

Text:

- [1] Zwiebach, Barton. A First Course in String Theory. New York: Cambridge UP, 2004. Print. Chapters 1-10
- [2] Kühnel, Wolfgang. Differential Geometry: Curves - Surfaces - Manifolds. Providence, RI: American Mathematical Society, 2006. Print.

Electronic:

- [3] Carberry, Emma, Kai Fung, David Glasser, Michael Nagle, and Nizam Ordulu. "Lecture Notes on Minimal Surfaces." MIT OCW. MIT, 17 Feb. 2005. Web. 21 Sept. 2016.
- [4] Beeson, Michael. "Notes on Minimal Surfaces." Michaelbeeson.com. N.p., 9 Aug. 2007. Web. 21 Sept. 2016.
- [5] Weber, Matthias. "Classical Minimal Surfaces in Euclidean Space by Examples." Indiana University Bloomington, 25 Sept. 2001. Web. 21 Sept. 2016.
- [6] Tong, David. "The Relativistic String." String Theory Lecture Notes. Cambridge University, n.d. Web. 21 Sept. 2016.
- [7] "Open Questions: Superstring Theory." Open Questions. Ed. Charles Daney. N.p., n.d. Web. 21 Sept. 2016.
- [8] Lerche, W. "Recent Developments in String Theory." Theoretical Physics Division. CERN, n.d. Web. 21 Sept. 2016.